SYMMETRY-INDUCED MODAL CHARACTERISTICS OF UNIFORM WAVEGUIDES

Paul R. McIsaac
School of Electrical Engineering
Cornell University
Ithaca, New York 14850

Abstract

The application of symmetry analysis to uniform waveguides is discussed. Symmetry analysis provides exact information concerning mode classification, mode degeneracy, modal field symmetries, and the minimum waveguide sectors which completely determine the modes in each mode class.

Introduction

Symmetry analysis can provide information about the general characteristics of the modes of microwave, millimeter, and optical waveguides, and it can suggest strategies to minimize the computer time required when a numerical analysis of the modes of a particular structure is undertaken. In this paper, symmetry analysis is applied to uniform waveguides which may be transversely inhomogeneous, but whose media are isotropic and piecewise-homogeneous.

Symmetry analysis, which is based on the theory of group representations, enables one to: classify all of the modes of a waveguide into mode classes based on the modal field symmetries, determine the possible mode degeneracies between mode classes, determine the azimuthal symmetries of the modal electromagnetic fields for each mode class, and specify the minimum sector of the waveguide cross section which can completely determine the modes of a given mode class. All of these results are obtained from a knowledge of the waveguide symmetry type, and solving a boundary value problem is not required.

Symmetry Analysis of Uniform Waveguides

Uniform waveguides belong to one of two families of symmetry types based on the spatial operations which leave the waveguide cross section unchanged in appearance. For the first family of symmetry types, the only symmetry operations are rotations about the waveguide axis. If the minimum rotation which leaves the cross section unchanged is $2\pi/n$ radians (n an integer), then there are a total of n different rotations which are symmetry operations: $2\pi/n$, $4\pi/n$, $6\pi/n$, - - -, $2(n-1)\pi/n$, and 2π radians (the last rotation is the identity operation). A waveguide with these symmetry operations is said to have C_n symmetry. Figure 1 shows the cross sections of several waveguides with C_n symmetry.

A waveguide may have reflection symmetry in a plane containing the waveguide axis in addition to rotation symmetry. If a waveguide with C_n symmetry has at least one such reflection plane, then it must have exactly n reflection planes, spaced azimuthally by π/n radians. A waveguide with both reflection and rotation symmetry operations is said to have C_{nv} symmetry. Figure 2 shows the cross sections of several waveguides with C_{nv} symmetry.

For a given waveguide symmetry type, C_n or C_{nv}, all of the modes can be classified into a number of mode classes with the total number of mode classes depending on the value of n in C_n or C_{nv}. The modal electromagnetic fields for all of the modes in the same mode class will have the same general azimuthal symmetry, although the detailed dependence on the azimuthal coordinate may differ. The major differences between the field patterns for different modes in the same mode class lie in their radial variations. The value of n also determines the number of mode classes with non-degenerate modes and the number of mode class pairs with mutually degenerate modes. Tables 1 and 2 show the number of mode classes and the mode degeneracies for waveguides with C_n and C_{nv} symmetries, respectively. These tables apply to the infinite number of modes which form the discrete mode spectrum of closed-boundary waveguides, as well as to the finite number of modes in the discrete mode spectrum and the continuous spectrum of modes in an open-boundary waveguide.

The various mode classes for waveguides of a particular symmetry type are distinguished by the azimuthal symmetry of their modal electromagnetic fields. That is, for any of the mode classes belonging to this symmetry type, one can describe the general azimuthal symmetry of the modal electric and magnetic fields. Further, based on the azimuthal symmetry of the electromagnetic fields of a mode class, one can specify a minimum sector of the waveguide cross section, together with associated boundary conditions for this sector, which is necessary and sufficient to completely determine all the modes of that mode class. This will be illustrated for two specific waveguides.

Figure 3a shows a closed-boundary waveguide with C_4 symmetry. This waveguide has a total of four mode classes; two of these are non-degenerate, while the other two mode classes form a pair with mutually degenerate modes. Figures 3b and 3d show the minimum waveguide sectors which completely determine the modes of the two non-degenerate mode classes. Here, dotted lines indicate periodic boundary conditions for the sector, and dot-dash lines indicate quasi-periodic boundary conditions (fields equal in magnitude, but reversed in sign). Figure 3c shows the minimum waveguide sector for the pair of degenerate mode classes. Solution of the appropriate boundary value problem in any of the minimum waveguide sectors shown will provide the modes belonging to the mode class associated with that sector.

Figure 4a shows an open-boundary waveguide which has seven dielectric rods (or fibers) in a close-packed structure with C_{6v} symmetry. This waveguide has a total of eight mode classes; of these, four are non-degenerate, and the other four form two pairs with each pair having mutually degenerate modes. Figures 4b, 4c, 4f, and 4g show the minimum waveguide sectors which completely determine the non-degenerate mode classes. Solid lines indicate a short-circuit boundary, while dashed lines indicate an open-circuit boundary. Figures 4d and 4e show the minimum waveguide sectors which completely determine the modes of the degenerate mode class pairs. Note that in this case, two figures are given for each degenerate mode class pair; thus the degeneracy is removed if these minimum waveguide sectors are used to determine the modes of each mode class. Again, solution of the appropriate boundary value problem in any of the minimum waveguide sectors shown will provide the modes belonging to the mode class associated with that sector.

Conclusion

These two examples illustrate some of the information that can be derived from symmetry analysis of uniform waveguides, based on the symmetry type of the structure under consideration. In particular, the cataloging of the modes into degenerate or non-degenerate mode classes was illustrated, and the minimum waveguide sectors for the mode classes were displayed. The use of the minimum waveguide sectors should enable an appreciable reduction in computation time in any numerical analysis of a symmetrical waveguide structure.

Table 1. Table of mode classes and mode degeneracies for uniform waveguides with C_n symmetry

n	Number of non-degenerate mode classes	Number of pairs of two-fold degenerate mode classes	Total number of mode classes
odd	1	(n-1)/2	n
even	2	(n-2)/2	n
∞	1	∞	∞

Table 2. Table of mode classes and mode degeneracies for uniform waveguides with C_{nv} symmetry

n	Number of non-degenerate mode classes	Number of pairs of two-fold degenerate mode classes	Total number of mode classes
odd	2	(n-1)/2	n+1
even	4	(n-2)/2	n+2
∞	2	∞	∞

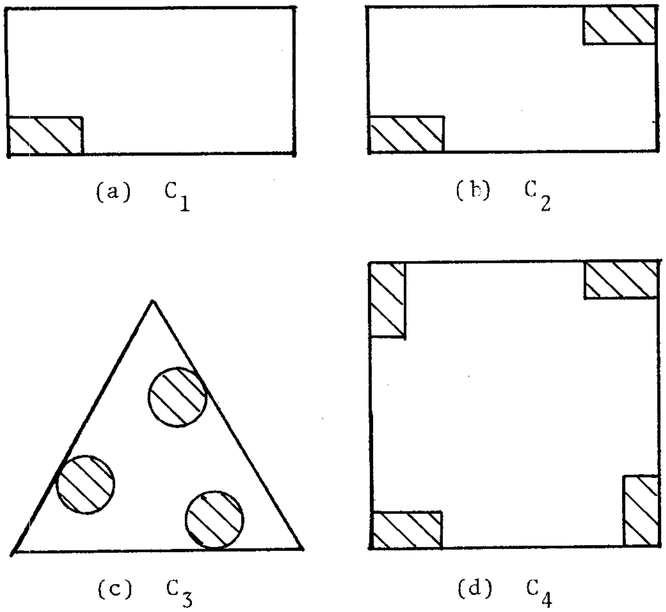

Fig. 1. Uniform waveguides with C_n symmetry.

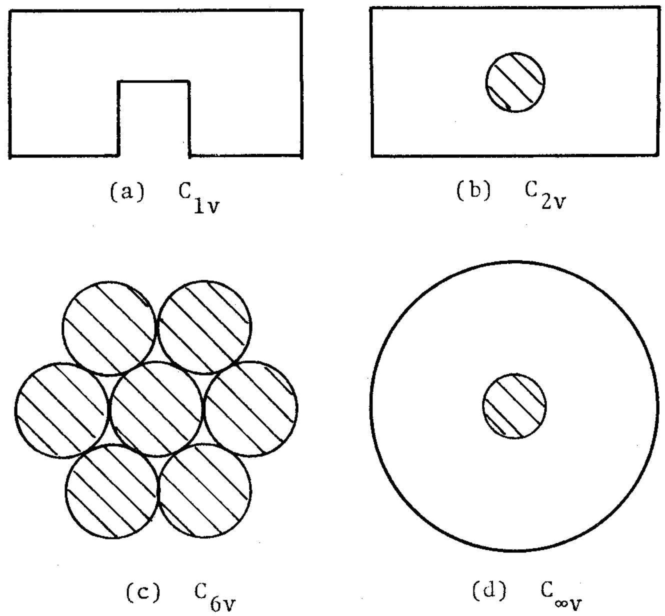

Fig. 2. Uniform waveguides with C_{nv} symmetry.

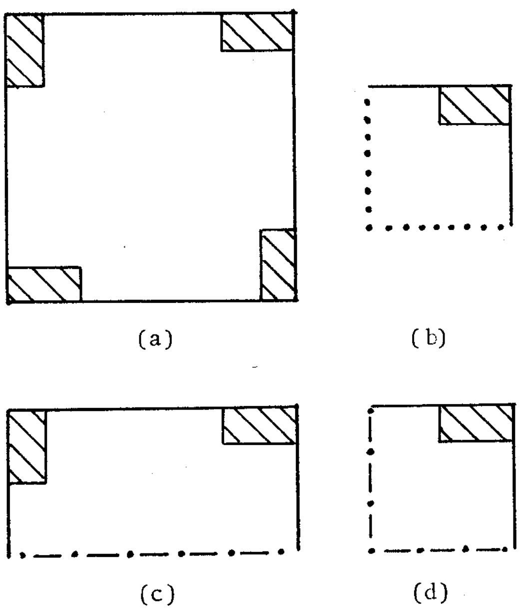

Fig. 3. Minimum waveguide sectors for a waveguide with C_4 symmetry.
(a) Waveguide with C_4 symmetry.
(b) First mode class.
(c) Second and third mode classes.
(d) Fourth mode class.

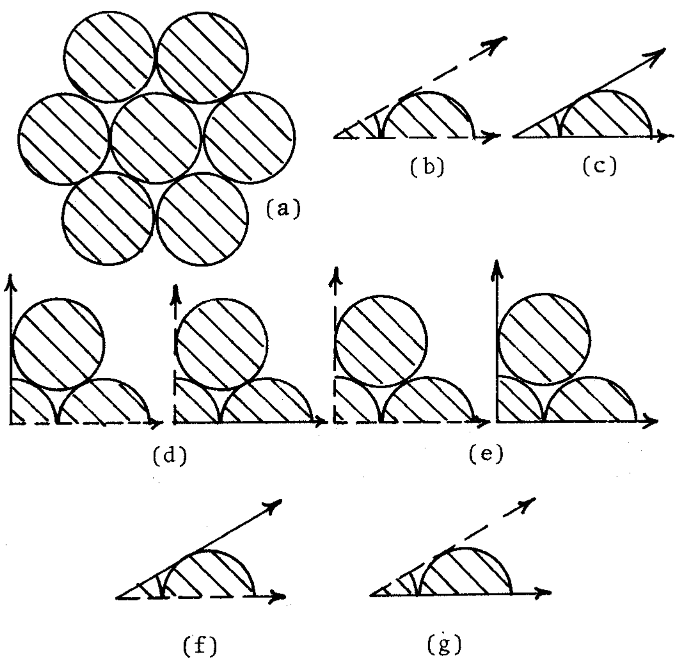

Fig. 4. Minimum waveguide sectors for a waveguide with C_{6v} symmetry.
(a) Waveguide with C_{6v} symmetry.
(b) First mode class.
(c) Second mode class.
(d) Third and fourth mode classes.
(e) Fifth and sixth mode classes.
(f) Seventh mode class.
(g) Eighth mode class.